ALGEBRA PROPERTIES

ARITHMETIC PROPERTIES

ASSOCIATIVE $a(bc) = (ab)c$

COMMUTATIVE $a + b = b + a$ and $ab = ba$

DISTRIBUTIVE $a(b + c) = ab + ac$

ARITHMETIC OPERATIONS EXAMPLES

$ab + ac = a(b + c)$

$a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$

$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

$\frac{a - b}{c - d} = \frac{b - a}{d - c}$

$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$

$\frac{ab + ac}{a} = b + c, a \neq 0$

$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$

QUADRATIC EQUATION

For the equation

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

RADICAL PROPERTIES

$a, b \geq 0$ for even n

$\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

$\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[n]{a^n} = a$, if n is odd

$\sqrt[n]{a^n} = |a|$, if n is even

LOGARITHM PROPERTIES

if $y = \log_b x$ then $b^y = x$

$\log_b b = 1$ and $\log_b 1 = 0$

$\log_b b^x = x$

$b^{\log_b x} = x$

$\log_a x = \frac{\log_b x}{\log_b a}$

$\log_b(x^r) = r \log_b x$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

EXPONENT PROPERTIES

$a^n a^m = a^{n+m}$

$(a^n)^m = a^{nm}$

$(ab)^n = a^n b^n$

$a^{-n} = \frac{1}{a^n}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$

$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}}$

$a^0 = 1, a \neq 0$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$\frac{1}{a^{-n}} = a^n$

$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}}$

COMMON FACTORING EXAMPLES

$x^2 - a^2 = (x + a)(x - a)$

$x^2 + 2ax + a^2 = (x + a)^2$

$x^2 - 2ax + a^2 = (x - a)^2$

$x^2 + (a + b)x + ab = (x + a)(x + b)$

$x^3 + 3ax^2 + 3a^2x + a^3 = (x + a)^3$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

PROPERTIES OF INEQUALITIES

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $a/c < b/c$

If $a < b$ and $c < 0$ then $ac > bc$ and $a/c > b/c$

PROPERTIES OF COMPLEX NUMBERS

$i = \sqrt{-1}$

$i^2 = -1$

$\sqrt{-a} = i\sqrt{a}, \quad a \geq 0$

$(a + bi) + (c + di) = a + c + (b + d)i$

$(a + bi) - (c + di) = a - c + (b - d)i$

$(a + bi)(c + di) = ac - bd + (ad + bc)i$

$(a + bi)(a - bi) = a^2 + b^2$

$|a + bi| = \sqrt{a^2 + b^2}$

$\overline{(a + bi)} = a - bi$

$\overline{(a + bi)}(a + bi) = |a + bi|^2$

$\frac{1}{(a + bi)} = \frac{(a - bi)}{(a + bi)(a - bi)} = \frac{a - bi}{a^2 + b^2}$

ABSOLUTE VALUE

$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$

$|a| = |-a|$

$|a| \geq 0$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a + b| \leq |a| + |b|$

COMPLETING THE SQUARE

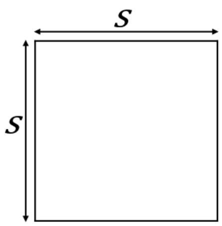
$ax^2 + bx + c = a(\ldots)^2 + \text{constant}$

1. Divide by the coefficient a.
2. Move the constant to the other side.
3. Take half of the coefficient b/a, square it and add it to both sides.
4. Factor the left side of the equation.
5. Use the square root property.
6. Solve for x.

GEOMETRY SHAPES AND SOLIDS

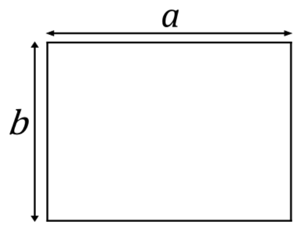
SQUARE

$P = 4s$
$A = s^2$



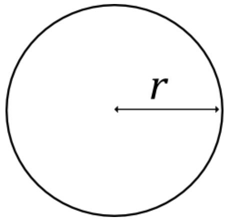
RECTANGLE

$P = 2a + 2b$
$A = ab$



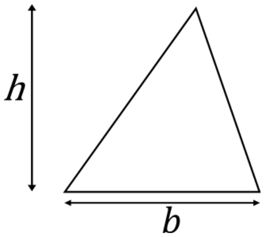
CIRCLE

$P = 2\pi r$
$A = \pi r^2$



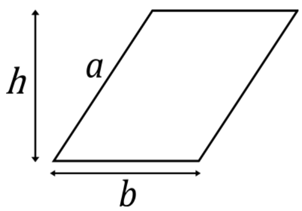
TRIANGLE

$P = a + b + c$
$A = \frac{1}{2}bh$



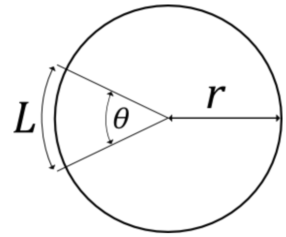
PARALLELOGRAM

$P = 2a + 2b$
$A = bh$



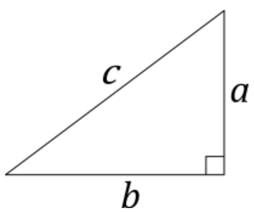
CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$
$A = \pi r^2 \frac{\theta}{360°}$



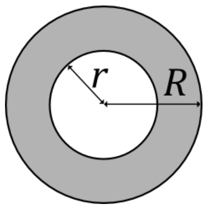
PYTHAGOREAN THEOREM

$a^2 + b^2 = c^2$
$c = \sqrt{a^2 + b^2}$



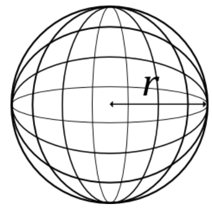
CIRCULAR RING

$A = \pi(R^2 - r^2)$



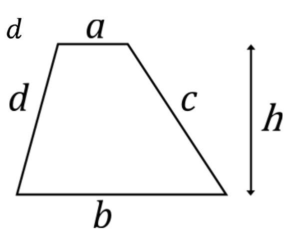
SPHERE

$S = 4\pi r^2$
$V = \frac{4\pi r^3}{3}$



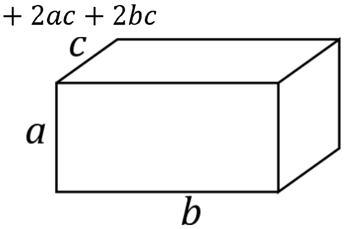
TRAPEZOID

$P = a + b + c + d$
$A = h\frac{a + b}{2}$



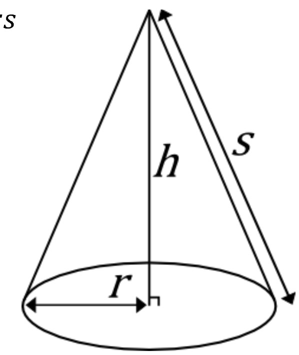
RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$
$V = abc$



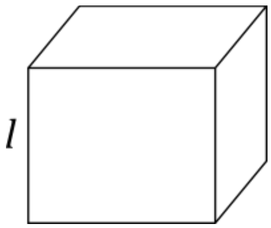
RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$
$s = \sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$



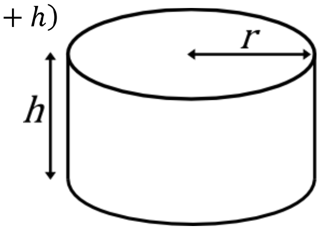
CUBE

$A = 6l^2$
$V = l^3$



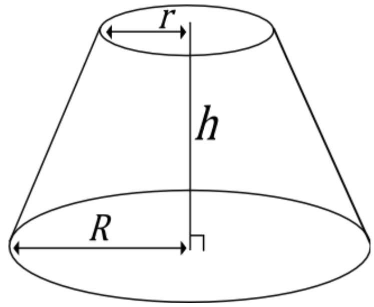
CYLINDER

$A = 2\pi r(r + h)$
$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$



TRIGONOMETRY LAWS AND IDENTITIES

TANGENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta$$

$$\tan(\theta + \pi n) = \tan\theta$$

$$\csc(\theta + 2\pi n) = \csc\theta$$

$$\sec(\theta + 2\pi n) = \sec\theta$$

$$\cot(\theta + \pi n) = \cot\theta$$

EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

PRODUCT TO SUM IDENTITIES

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

SUM TO PRODUCT IDENTITIES

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

LAW OF TANGENTS

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

MOLLWEIDE'S FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

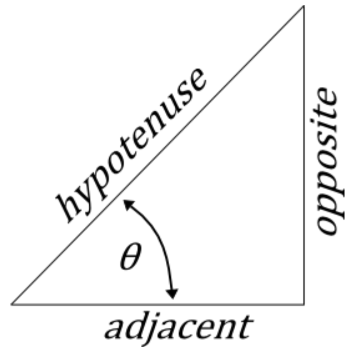
TRIGONOMETRY DEFINITION

RIGHT TRIANGLE DEFINITION

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\quad\csc\theta = \dfrac{hypotenuse}{opposite}$

$\cos\theta = \dfrac{adjacent}{hypotenuse}$ $\quad\sec\theta = \dfrac{hypotenuse}{adjacent}$

$\tan\theta = \dfrac{opposite}{adjacent}$ $\quad\cot\theta = \dfrac{adjacent}{opposite}$



TRIG FUNCTIONS RANGE

$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

$-\infty \le \tan\theta \le \infty$

$\csc\theta \ge 1$ and $\csc\theta \le -1$

$\sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty \le \cot\theta \le \infty$

UNIT CIRCLE DEFINITION

$\sin\theta = y$

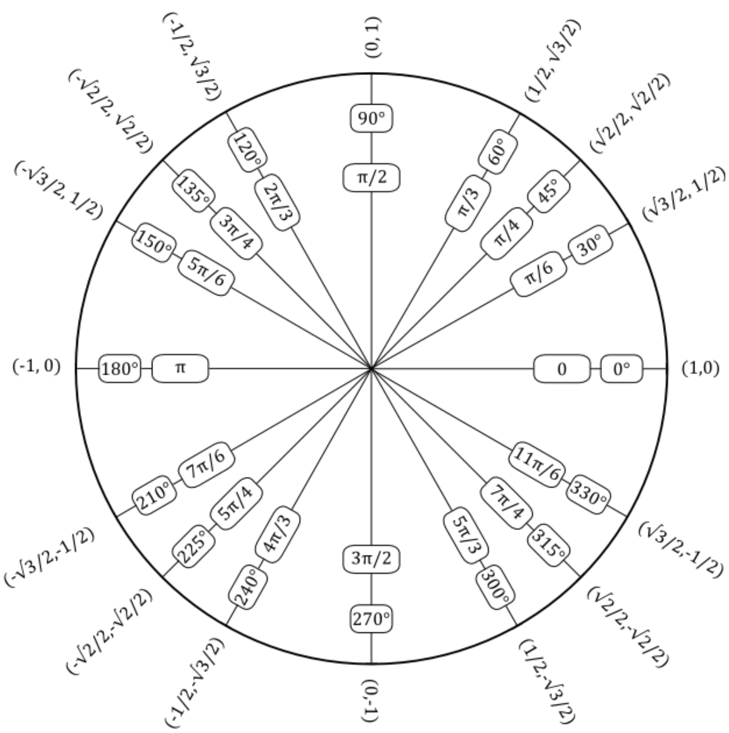
$\cos\theta = x$

$\tan\theta = \dfrac{y}{x}$

$\csc\theta = \dfrac{1}{y}$

$\sec\theta = \dfrac{1}{x}$

$\cot\theta = \dfrac{x}{y}$



TRIG FUNCTIONS DOMAIN

$\sin\theta,\ \ \theta$ can be any angle

$\cos\theta,\ \ \theta$ can be any angle

$\tan\theta,\ \ \theta \neq \left(n+\dfrac{1}{2}\right)\pi, \qquad n = 0, \pm1, \pm2, \ldots$

$\csc\theta,\ \ \theta \neq n\pi, \qquad n = 0, \pm1, \pm2, \ldots$

$\sec\theta,\ \ \theta \neq \left(n+\dfrac{1}{2}\right)\pi, \qquad n = 0, \pm1, \pm2, \ldots$

$\cot\theta,\ \ \theta \neq n\pi, \qquad n = 0, \pm1, \pm2, \ldots$

TRIG FUNCTIONS PERIOD

$\sin(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\cos(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\tan(\omega\theta) \rightarrow T = \dfrac{\pi}{\omega}$

$\csc(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\sec(\omega\theta) \rightarrow T = \dfrac{2\pi}{\omega}$

$\cot(\omega\theta) \rightarrow T = \dfrac{\pi}{\omega}$

INVERSE TRIG FUNCTION NOTATION

$\sin^{-1}x \equiv \arcsin x \equiv \text{Asin}\, x$

$\cos^{-1}x \equiv \arccos x \equiv \text{Acos}\, x$

$\tan^{-1}x \equiv \arctan x \equiv \text{Atan}\, x$

INVERSE TRIG DOMAIN

$\sin^{-1}x : -1 \le x \le 1$

$\cos^{-1}x : -1 \le x \le 1$

$\tan^{-1}x : -\infty \le x \le \infty$

INVERSE TRIG FUNCTION RANGE

$-\dfrac{\pi}{2} \le \sin^{-1}x \le \dfrac{\pi}{2}$

$0 \le \cos^{-1}x \le \pi$

$-\dfrac{\pi}{2} \le \tan^{-1}x \le \dfrac{\pi}{2}$

CALCULUS DERIVATIVES AND LIMITS

DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$

$$(f(x)\pm g(x))' = f'(x)\pm g'(x)$$

$$\frac{d}{dx}(c) = 0$$

MEAN VALUE THEOREM

If f *is* differentiable on the interval (a,b) and continuous at the end points there exists a c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}\big(f(g(x))\big) = f'(g(x))g'(x)$$

COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x\tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x\cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x) = a^x\ln(a)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x>0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$$

CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

$$\frac{d}{dx}\left(f(x)^{g(x)}\right) = f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))\,g'(x)\right)$$

PROPERTIES OF LIMITS

These properties require that the limit of $f(x)$ and $g(x)$ exist

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a}f(x)$$

$$\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a}f(x)\lim_{x\to a}g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)} \text{ if } \lim_{x\to a}g(x)\neq 0$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a}f(x)\right]^n$$

LIMIT EVALUATION METHOD – FACTOR AND CANCEL

$$\lim_{x\to -3}\frac{x^2-x-12}{x^2+3x} = \lim_{x\to -3}\frac{(x+3)(x-4)}{x(x+3)} = \lim_{x\to -3}\frac{(x-4)}{x} = \frac{7}{3}$$

L'HOPITAL'S RULE

$$\text{If } \lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

LIMIT EVALUATIONS AT +-∞

$\lim_{x\to\infty} e^x = \infty$ and $\lim_{x\to-\infty} e^x = 0$

$\lim_{x\to\infty}\ln(x) = \infty$ and $\lim_{x\to 0^+}\ln(x) = -\infty$

If $r>0$ then $\lim_{x\to\infty}\frac{c}{x^r} = 0$

If $r>0$ & x^r is real for $x<0$ then $\lim_{x\to-\infty}\frac{c}{x^r} = 0$

$\lim_{x\to\pm\infty} x^r = \infty$ for even r

$\lim_{x\to\infty} x^r = \infty$ & $\lim_{x\to-\infty} x^r = -\infty$ for odd r

CALCULUS INTEGRALS

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a,b]$ and $F' = f$

INTEGRATION PROPERTIES

$$\int_a^b cf(x)dx = c\int_a^b f(x)dx$$

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^a f(x)dx = 0 \text{ and } \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

COMMON INTEGRALS

$$\int k\ dx = kx + C$$

$$\int x^n\ dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

$$\int x^{-1}\ dx = \int \frac{1}{x}\ dx = \ln|x| + C$$

$$\int \frac{1}{ax+b}\ dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int \ln(x)\ dx = x\ln(x) - x + C$$

$$\int e^x\ dx = e^x + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sin x\ dx = -\cos x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \tan x\ dx = \ln|\sec x| + C$$

$$\int \sec x\ dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2}\ dx = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}}\ dx = \sin^{-1}\left(\frac{u}{a}\right) + C$$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x \sum_{k=0}^{n-1} f(x_k) \qquad R_n = \Delta x \sum_{k=1}^{n} f(x_k)$$

Midpoint Rule

$$M_n = \Delta x \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n)\right)$$

TRIGNOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$ $dx = a\cos\theta\, d\theta$	$\sqrt{a^2 - a^2\sin^2\theta}$ $= a\cos\theta$	$1 - \sin^2\theta$ $= \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$ $dx = a\sec\theta\tan\theta\, d\theta$	$\sqrt{a^2\sec^2\theta - a^2}$ $= a\tan\theta$	$\sec^2\theta - 1$ $= \tan^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$ $dx = a\sec^2\theta\, d\theta$	$\sqrt{a^2 + a^2\tan^2\theta}$ $= a\sec\theta$	$1 + \tan^2\theta$ $= \sec^2\theta$

APPROXIMATION BY SIMPSON RULE FOR EVEN N

$$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x))\, g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$ and $du = g'(x)dx$

INTEGRATION BY PARTS

$$\int u\ dv = uv - \int v\ du \quad \text{where } v = \int dv$$

or

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$